

Recursions on Strings

Palindrome

"racecar"

"aracecars"

"raceacar"

Reversal

"abcd"

Number of Occurrences

"abca"

'a'

'b'

Problem: Palindrome

```
boolean isPalindrome(String word) {  
    if(word.length() == 0 || word.length() == 1) {  
        /* base case */  
        return true;  
    }  
    else {  
        /* recursive case */  
        char firstChar = word.charAt(0);  
        char lastChar = word.charAt(word.length() - 1);  
        String middle = word.substring(1, word.length() - 1);  
        return  
            firstChar == lastChar  
            /* See the API of java.lang.String.substring. */  
            && isPalindrome(middle);  
    }  
}
```

Problem: Reverse of a String

```
String reverseOf (String s) {  
    if(s.isEmpty()) { /* base case 1 */  
        return "";  
    }  
    else if(s.length() == 1) { /* base case 2 */  
        return s;  
    }  
    else { /* recursive case */  
        String tail = s.substring(1, s.length());  
        String reverseOfTail = reverseOf (tail);  
        char head = s.charAt(0);  
        return reverseOfTail + head;  
    }  
}
```

Problem: Number of Occurrences

```
int occurrencesOf (String s, char c) {  
    if(s.isEmpty()) {  
        /* Base Case */  
        return 0;  
    }  
    else {  
        /* Recursive Case */  
        char head = s.charAt(0);  
        String tail = s.substring(1, s.length());  
        if(head == c) {  
            return 1 + occurrencesOf (tail, c);  
        }  
        else {  
            return 0 + occurrencesOf (tail, c);  
        }  
    }  
}
```

Recursion on an Array: Passing **new Sub-Arrays**

```
void m(int[] a) {  
    if(a.length == 0) { /* base case */ }  
    else if(a.length == 1) { /* base case */ }  
    else {  
        int[] sub = new int[a.length - 1];  
        for(int i = 1; i < a.length; i++) { sub[i - 1] = a[i]; }  
        m(sub) } }  
}
```

Say $a_1 = \{\}$, consider $m(a_1)$

Say $a_2 = \{A, B, C\}$, consider $m(a_2)$

Recursion on an Array: Passing Same Array Reference

```
void m(int[] a, int from, int to) {  
    if(from > to) { /* base case */ }  
    else if(from == to) { /* base case */ }  
    else { m(a, from + 1, to) } }
```

Say $a1 = \{\}$, consider $m(a1, 0, a1.length - 1)$

Say $a2 = \{A, B, C\}$, consider $m(a2, 0, a2.length - 1)$

Problem: Are All Numbers Positive?

Say $a = \{\}$

Say $a = \{4\}$

Say $a = \{4, 7, 3, 9\}$

Say $a = \{5, 3, -2, 9\}$

Problem: Are All Numbers Positive?

```
boolean allPositive(int[] a) {  
    return allPositiveHelper(a, 0, a.length - 1);  
}  
  
boolean allPositiveHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return a[from] > 0;  
    }  
    else { /* recursive case */  
        return a[from] > 0 && allPositiveHelper(a, from + 1, to);  
    }  
}
```


Tracing Recursion: `allPositive`

Say `a = {}`

`allPositive(a)`

`allPH(a,0,-1)`

```
boolean allPositive(int[] a) {  
    return allPositiveHelper(a, 0, a.length - 1);  
}  
  
boolean allPositiveHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return a[from] > 0;  
    }  
    else { /* recursive case */  
        return a[from] > 0 && allPositiveHelper(a, from + 1, to);  
    }  
}
```

Tracing Recursion: allPositive

Say a = {4}

allPositive(a)

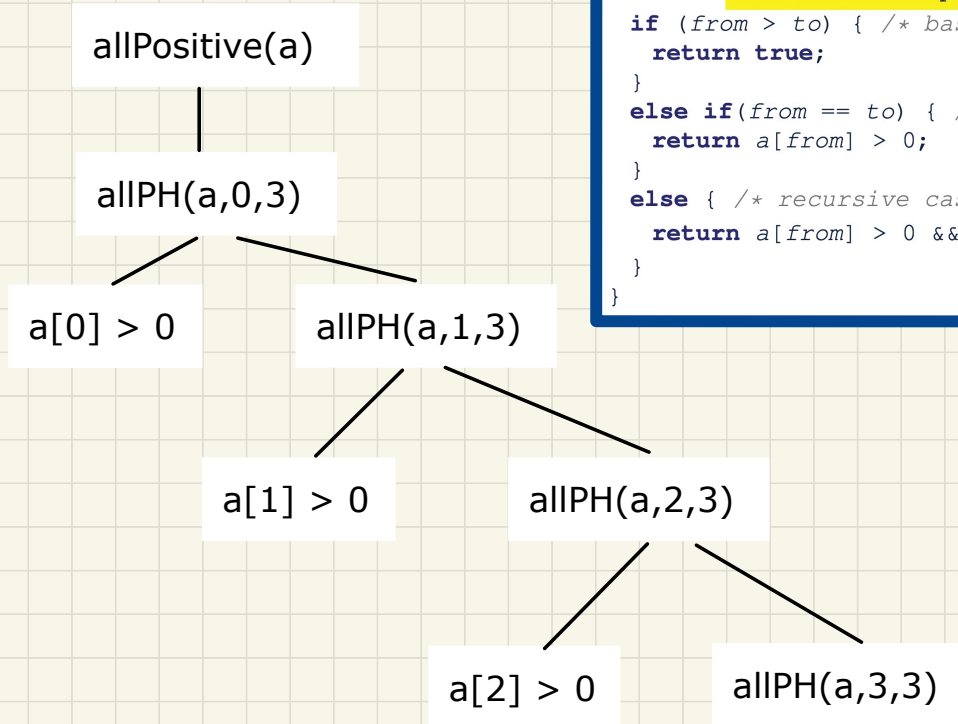
allPH(a,0,0)

a[0] > 0

```
boolean allPositive(int[] a) {  
    return allPositiveHelper(a, 0, a.length - 1);  
}  
  
boolean allPositiveHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return a[from] > 0;  
    }  
    else { /* recursive case */  
        return a[from] > 0 && allPositiveHelper(a, from + 1, to);  
    }  
}
```

Tracing Recursion: **allPositive**

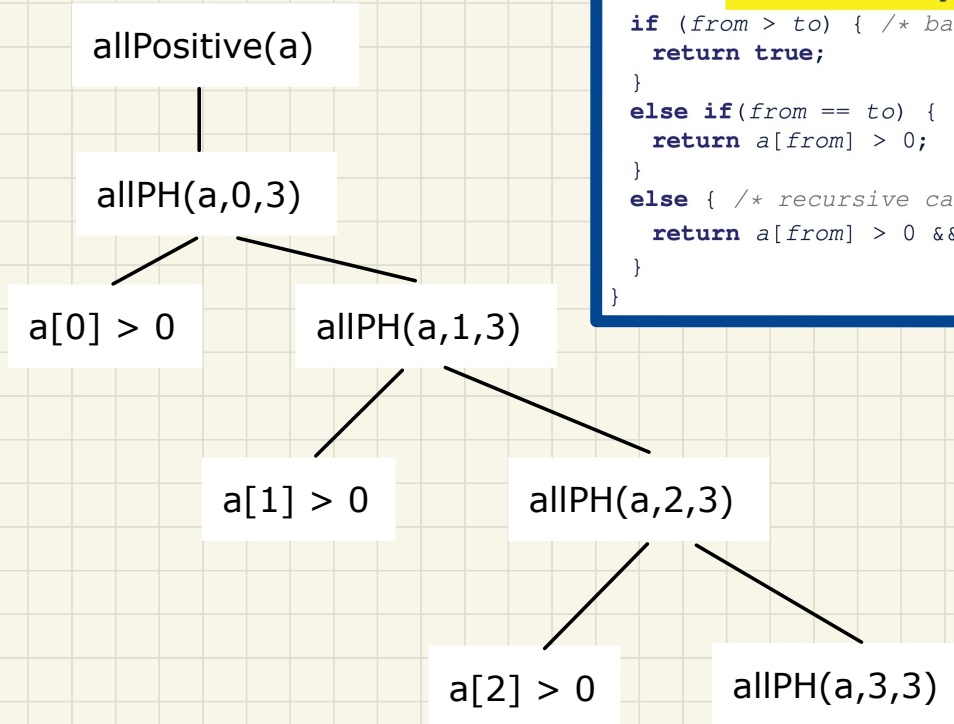
Say $a = \{4, 7, 3, 9\}$



```
boolean allPositive(int[] a) {  
    return allPositiveHelper(a, 0, a.length - 1);  
}  
  
boolean allPositiveHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return a[from] > 0;  
    }  
    else { /* recursive case */  
        return a[from] > 0 && allPositiveHelper(a, from + 1, to);  
    }  
}
```

Tracing Recursion: **allPositive**

Say $a = \{5, 3, -2, 9\}$



```
boolean allPositive(int[] a) {  
    return allPositiveHelper(a, 0, a.length - 1);  
}  
  
boolean allPositiveHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return a[from] > 0;  
    }  
    else { /* recursive case */  
        return a[from] > 0 && allPositiveHelper(a, from + 1, to);  
    }  
}
```

Problem: Are Numbers Sorted?

Say $a = \{\}$

Say $a = \{4\}$

Say $a = \{3, 6, 6, 7\}$

Say $a = \{3, 6, 5, 7\}$

Problem: Are Numbers Sorted?

```
boolean isSorted(int[] a) {  
    return isSortedHelper(a, 0, a.length - 1);  
}  
  
boolean isSortedHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return true;  
    }  
    else {  
        return a[from] <= a[from + 1]  
            && isSortedHelper(a, from + 1, to);  
    }  
}
```

Tracing Recursion: **isSorted**

Say $a = \{\}$

isSorted(a)

isSH(a,0,-1)

```
boolean isSorted(int[] a) {  
    return isSortedHelper(a, 0, a.length - 1);  
}  
  
boolean isSortedHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return true;  
    }  
    else {  
        return a[from] <= a[from + 1]  
            && isSortedHelper(a, from + 1, to);  
    }  
}
```

Tracing Recursion: **isSorted**

Say $a = \{4\}$

isSorted(a)

isSH(a,0,0)

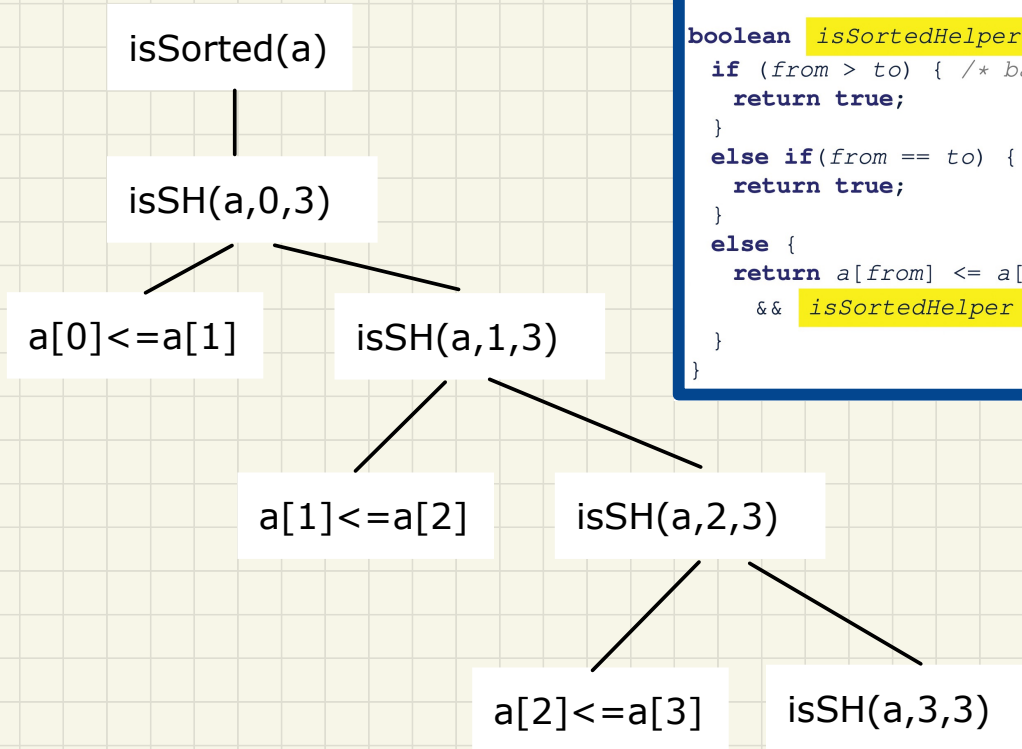
return **true**

```
boolean isSorted(int[] a) {  
    return isSortedHelper(a, 0, a.length - 1);  
}  
  
boolean isSortedHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return true;  
    }  
    else {  
        return a[from] <= a[from + 1]  
            && isSortedHelper(a, from + 1, to);  
    }  
}
```


Tracing Recursion: **isSorted**

Say $a = \{3, 6, 6, 7\}$

```
boolean isSorted(int[] a) {  
    return isSortedHelper(a, 0, a.length - 1);  
}  
  
boolean isSortedHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return true;  
    }  
    else {  
        return a[from] <= a[from + 1]  
            && isSortedHelper(a, from + 1, to);  
    }  
}
```



Tracing Recursion: **isSorted**

Say $a = \{3, 6, 5, 7\}$

```
boolean isSorted(int[] a) {  
    return isSortedHelper(a, 0, a.length - 1);  
}  
  
boolean isSortedHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return true;  
    }  
    else {  
        return a[from] <= a[from + 1]  
            && isSortedHelper(a, from + 1, to);  
    }  
}
```

